Tentamen Quantum Fysica I
August 17, 1999

Please **print** *your name, student number and complete address on the first page. Each problem is to be answered on a* separate *page. Print your name on top of each page.*

Elke opgave op en apart vel. Zet op het eerste vel duidelijk uw naam, student nummer en adres. Op elk volgend vel uw naam vermelden.

Problem 1.

For each of the parts below, write the *most general* time-independent wave function in coordinate space, that is consistent with the conditions mentioned in that part. (*Note*: Do not bother about the normalization.)

(a) A particle in one dimension whose momentum is p_0

(b) A particle in one dimension located exactly at x_0.

(c) A particle in one dimension whose momentum lies within $p_1 < p < p_2$.

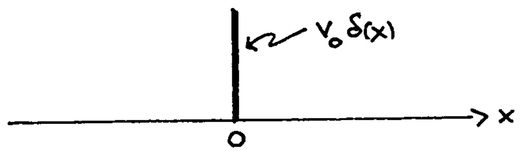
(d) A particle in one dimension whose wave function has parity -1.

Problem 2.

A particle of mass m moves in one dimension where the only potential $V(x) = V_0\delta(x)$ is at the origin ($V_0 > 0$). A free particle of energy $E > 0$ approaches the origin from the left.

(a) What is the wave vector k for the incoming particle?

(b) Derive an expression for the amplitude T of the transmitted wave as a function of k, V_0, m and $\hbar$.



Problem 3.

Let $u_n(x)$ denote the orthonormal stationary states of a system corresponding to energy E_n. Let $\psi(x,0)$ denote the normalized wave function of the system at time $t=0$. Suppose $\psi(x,0)$ to be such that a measurement of the energy of the system would yield the value E_1 with probability $\frac{1}{2}$, E_2 with probability $\frac{1}{4}$, and E_3 with probability $\frac{1}{4}$.

(a) Write the most general expression you can for $\psi(x,0)$ in terms of the $u_n(x)$ consistent with the given data.

(b) Write the expression for the state function $\psi(x,t)$ at time $t>0$.

(c) Which of the following quantities have expectation values which are *independent* of the time for this system when it is in the $\psi(x,t)$ of part (b)?

(i) position, $<x>$.

(ii) kinetic energy, $\frac{p^2}{2m}$.

(iii) potential energy, $V(x)$.

(iv) Hamiltonian, $<H>$

(d) Repeat part (c) when the system is in one of its stationary states.

Problem 4.

WOULD NOT BE AN EXAM QUESTION IN QF-I 1999/2000.

Let x_0 and p_0 denote the expectation values of x and p for the state $\phi_0(x)$. Consider the state

$$\psi(x) = e^{-p_0 x/\hbar}\phi(x_0+x).$$

(a) Evaluate the expectation value $<\psi|x|\psi>$.

(b) Evaluate the expectation value $<\psi|p|\psi>$.

(c) Does your result for (a) and (b) violate the uncertainty principle? Explain.

Problem 5.

A particle moving in one dimension has a first-excited-state eigenfunction associated with the with the energy E_1 given by

$$u_1(x) = xu_0(x)$$

where u_0 is the ground-state eigenfunction associated with the energy eigenvalue E_0. Given that the potential vanishes at $x=0$

(a) Determine the ratio E_1/E_0.

(b) What is the potential $V(x)$?

Hint: Simple manipulation allows one to get a first-order differential equation for u_0. Solve for u_0 and proceed to determine $V(x)$.